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Similarity degrees and uncertainty measures in intuitionistic fuzzy decision tables

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Abstract. Recently, making decisions and analyzing data are getting more and more attention by taking advantage of rough set and intuitionistic fuzzy set theories. Additionally, it can be found that many works have been developed about intuitionistic fuzzy rough set approaches from different viewpoints. In this article, we introduce similarity degrees and four kinds of uncertainty measurement, called θ-conditional entropies, θ-similarity intuitionistic fuzzy accuracies, θ-similarity intuitionistic fuzzy roughness and θ-rough decision entropies in intuitionistic fuzzy decision tables. Also, we provide a novel method for classifying the objects' intuitionistic fuzzy decision table. Moreover, we carefully discuss the lower approximation and upper approximation of a given set and classify their important properties based on the novel classes in the intuitionistic fuzzy decision table. Furthermore, an illustrated example is employed to demonstrate the conceptual arguments of these measurements based on different similarity degrees and similarity rates. From this, it can be found that the new measures are superior to the classical accuracy and roughness, and the method is valuable and useful in real life situations.

Keywords: Classifications, decision information table, intuitionistic fuzzy set, rough set, similarity measures

1. Introduction

Pawlak proposed the rough set theory [4]. This theory is a development of the classical set. Now, approximations may be generalized by making use of non-equivalence relations [16]. More and more success can be obtained in knowledge acquisition and reasoning in incomplete information systems by using the extensions of the traditional rough set mode. However, it is common to meet data with fuzzy values in real life. So, fuzzy set theory was proposed by Zadeh, which is commonly made use of when describing quantitative data expressed and membership functions in linguistic terms and intelligent information systems. Many researchers have extended rough set theory to fuzzy situations. Some achievements of these studies have moved to the introductions of concepts combining fuzzy rough sets and rough fuzzy sets.

Uncertainty measures are very important in rough set theory nowadays. In general, researchers apply roughness to accurately evaluate the uncertainty of some sets. However, they are not very useful to characterize the uncertainty of a given table. These two kinds of measures are both based on a pair of approximations, which are lower approximation and upper approximation. Entropy theory is a highly helpful tool for representing information contents from different formats and has been applied in many areas.

Many excellent achievements regarding entropy and uncertainty measures have been received. Many measures-based information theories have been presented and applied to quantify relationships between attributes and the importance of attributes in various fields. Yao [19] studied approximation structures and hierarchical granulation, and obtained some

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important results in a stratified rough set model. Liang et al. [7] proposed an axiomatic formal about knowledge granulation for an information system. Xu et al. [17] investigated the definitions and some important properties of knowledge uncertainty measures and information entropy in ordered information systems.

In the real world, some attribute values are pairs of fuzzy values for an object. For this reason, Atanassov [8] first proposed the intuitionistic fuzzy set model in 1986, which is very effective when handling vagueness. Moreover, this theory is of great help to perform research into uncertainty theories. In fact, the intuitionistic fuzzy set model can be seen as a development of the classical fuzzy set by introducing non-membership and membership [12, 15]. Clearly, from the point of representing the indistinctions about some information, an intuitionistic fuzzy set model is more precise than traditional fuzzy set theory. Hence, it is a novel model to discuss uncertain information by integrating rough set theory with intuitionistic fuzzy set theory. This has come to be an interesting topic, and some achievements can be found in recent literature [1, 4, 13, 21].

Associated with intuitionistic fuzzy set theory [7, 14], the application fields of rough set theory have become more and more wide. For example, Zhou et al. [9] studied the rough approximation properties of Atanassov IF sets in fuzzy and crisp approximation spaces in which both axiomatic and constructive approaches are considered. Zhang et al. [18] provided a new notion of intuitionistic fuzzy rough sets by analyzing their important properties based on general binary relations, implication operators, and two universes. The interval-valued intuitionistic fuzzy rough approach is proposed by combing rough set theory with the interval-valued intuitionistic fuzzy set [20]. What's more, Xu [23] explored the similarity measures of the intuitionistic fuzzy set. Also, the intuitionistic fuzzy set model has recently become more and more important in dealing with uncertain problems. In intuitionistic fuzzy information systems, research about classifying objects based on the similarity degree is hard to find, and this issue is very important in dealing with data and information.

The structure of this article is arranged as follows. Firstly, in Section 2, we review some necessary and basic notations of rough set and intuitionistic fuzzy set models. In Section 3, six kinds of similarity degrees are considered, and the lower approximation and upper approximation of a given set are discussed based on the new similarity classes. Furthermore, we also explore their properties. In Section 4, some basic concepts and properties of uncertainty measurement in intuitionistic fuzzy decision tables, including θ-similarity intuitionistic fuzzy accuracies, θ-similarity intuitionistic fuzzy roughness, θ-conditional entropies, and θ-rough decision entropies are investigated. In section 5, we give a comparison with clustering methods. What's more, some experiments are studied to verify the effectiveness of the considered measures in section 6. Lastly, we conclude the article by providing a summary and giving ideas for further research work.

2. Preliminary

According to their attribute values, the concept of the information table presents a handy tool in describing objects. Generally, an information table is an ordinal triple $I = (U, A, f)$, in which U is the objects set (a non-empty finite set), A is a conditional attribute set and f is the relationship between U and A. With respect to equivalence relation R_A induced by attribute set A, the Pawlak's classical lower approximation and upper approximation of X can be found as follows [22].

$$\overline{R_A}(X) = \{u \in U | [u]_A \cap X \neq \emptyset\}$$

$$\underline{R_A}(X) = \{u \in U | [u]_A \subseteq X\}.$$

where $[u]_A$ is an equivalence class of u onto R_A. The set X is called a definable set if and only if $\underline{R_A}(X) = \overline{R_A}(X)$, Otherwise, set X is referenced as a rough set.

Intuitionistic fuzzy set theory has had a great effect on uncertainty theories. In the next section, the necessary notions about intuitionistic fuzzy sets will be recalled.

Definition 1. (See [8]) Let U be a universe (a non-empty finite set). An IF (intuitionistic fuzzy set) $\tilde{A}$ on U is an object having the following property:

$$\tilde{A} = \{< u, \mu f_{\tilde{A}}(u), \nu f_{\tilde{A}}(u) > | u \in U\}.$$

where $\nu f_{\tilde{A}} : U \rightarrow [0, 1]$ and $\mu f_{\tilde{A}} : U \rightarrow [0, 1]$ satisfy $0 \leq \mu f_{\tilde{A}}(u) + \nu f_{\tilde{A}}(u) \leq 1$ for all $u \in U$.

The functions $\nu f_{\tilde{A}}(u)$ and $\mu f_{\tilde{A}}(u)$ are, respectively, called the degrees of non-membership and membership of the element $u \in U$ to $\tilde{A}$ [8].

The degrees of membership and non-membership of object u in IF (intuitionistic fuzzy set) set $\tilde{A}$ are a pair $(\mu f_{\tilde{A}}(u), \nu f_{\tilde{A}}(u))$, and the hesitancy degree of u is $\pi f_{\tilde{A}}(u) = 1 - \mu f_{\tilde{A}}(u) - \nu f_{\tilde{A}}(u)$. Thus, the maximal possible degree of membership of u is $\mu f_{\tilde{A}}(u) + \pi f_{\tilde{A}}(u) + \nu f_{\tilde{A}}(u) = 1$. Thus, the possible degree of membership can also be presented in the form of interval $[\mu f_{\tilde{A}}(u), 1 - \nu f_{\tilde{A}}(u)]$.

So, $\varphi f(u) = \frac{\mu f_{\tilde{A}}(u)+1-\nu f_{\tilde{A}}(u)}{2}$ can be called the mean degree of membership.

Let $IF(U)$ denote the family of all IFs on U.

Let $\mathcal{I} = (U, A \cup \{d\})$ denote an intuitionistic fuzzy decision table (IFDT), where U is a domain (a non-empty finite set), A is a condition attributes set (a non-empty finite set) and d is a class label or decision attribute. For any $a_k \in A$, the pair $(\mu f_{a_k}(u_i), \nu f_{a_k}(u_i))$ stands for the degrees of membership and non-membership of object u_i on attribute a_k. Also,

$$\pi f_{a_k}(u_i) = 1 - \mu f_{a_k}(u_i) - \nu f_{a_k}(u_i) \tag{1}$$

gives the hesitancy degree of object u_i on attribute a_k. Moreover,

$$\varphi f_{a_k}(u_i) = \frac{\mu f_{a_k}(u_i) + 1 - \nu f_{a_k}(u_i)}{2} \tag{2}$$

expresses the degree of membership of object u_i on attribute a_k.

3. IFDT similarity degrees

For an intuitionistic fuzzy decision table, an object has degrees of membership and non-membership based on a particular attribute, so it is very difficult to classify the object. The similarity degree provides a good tool for this work.

In order to measure the similarity degree between two IFs $\tilde{A}_1, \tilde{A}_2$ on U, Fan [5] has provided the following general expression:

$$\begin{aligned} T(\tilde{A}_1, \tilde{A}_2) &= \frac{1}{n}\sum_{i=1}^{n}(1 - w_1 \left|\mu f_{\tilde{A}_1}(u_i) - \mu f_{\tilde{A}_2}(u_i)\right| \\ &\quad - w_2 \left|\nu f_{\tilde{A}_1}(u_i) - \nu f_{\tilde{A}_2}(u_i)\right| \\ &\quad - w_3 \left|\pi f_{\tilde{A}_1}(u_i) - \pi f_{\tilde{A}_2}(u_i)\right| \\ &\quad - w_4 \left|\varphi f_{\tilde{A}_1}(u_i) - \varphi f_{\tilde{A}_2}(u_i)\right|), \end{aligned}$$

where w_1, w_2, w_3, w_4 are the importance degree of each index, and they all satisfy the similarity degree $T(\tilde{A}_1, \tilde{A}_2) \geq 0$.

Case 1: If we let $w_1 = w_2 = w_3 = 0$, $w_4 = 1$ and pay close attention to the mean degree of membership, then we can gain the similarity degree [11] of $\tilde{A}_1, \tilde{A}_2$ as follows:

$$T_1(\tilde{A}_1, \tilde{A}_2) = \frac{1}{n}\sum_{i=1}^{n}(1 - \left|\varphi f_{\tilde{A}_1}(u_i) - \varphi f_{\tilde{A}_2}(u_i)\right|).$$

Case 2: If we let $w_1 = w_2 = \frac{1}{2}$, $w_3 = w_4 = 0$ and mainly focus on the degrees of non-membership and membership, then we can obtain the similarity degree [2] of $\tilde{A}_1, \tilde{A}_2$ as follows:

$$T_2(\tilde{A}_1, \tilde{A}_2) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\left|\mu f_{\tilde{A}_1}(u_i) - \mu f_{\tilde{A}_2}(u_i)\right| + \left|\nu f_{\tilde{A}_1}(u_i) - \nu f_{\tilde{A}_2}(u_i)\right|}{2}\right)$$

Case 3: If we let $w_1 = w_2 = \frac{1}{4}$, $w_3 = \frac{1}{2}$, $w_4 = 0$ and primarily emphasize the mean degree of membership based on the degrees of non-membership and membership, then we can obtain the similarity degree [3] of $\tilde{A}_1, \tilde{A}_2$ as follows:

$$\begin{aligned} T_3(\tilde{A}_1, \tilde{A}_2) &= \frac{T_1(\tilde{A}_1, \tilde{A}_2) + T_2(\tilde{A}_1, \tilde{A}_2)}{2} \\ &= \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\left|\mu f_{\tilde{A}_1}(u_i) - \mu f_{\tilde{A}_2}(u_i)\right| + \left|\nu f_{\tilde{A}_1}(u_i) - \nu f_{\tilde{A}_2}(u_i)\right|}{4} - \frac{\left|\varphi f_{\tilde{A}_1}(u_i) - \varphi f_{\tilde{A}_2}(u_i)\right|}{2}\right) \end{aligned}$$

Case 4: If we let $w_1 = w_2 = w_3 = \frac{1}{3}$, $w_4 = 0$ and treat the mean degree of membership and the degrees of non-membership and membership equally, then we can acquire the similarity degree [5] of $\tilde{A}_1, \tilde{A}_2$ as follows:

$$\begin{aligned} &T_4(\tilde{A}_1, \tilde{A}_2) \\ &= \frac{1}{n}\sum_{i=1}^{n}(1 - \frac{1}{3}(\left|\mu f_{\tilde{A}_1}(u_i) - \mu f_{\tilde{A}_2}(u_i)\right| \\ &\quad + \left|\nu f_{\tilde{A}_1}(u_i) - \nu f_{\tilde{A}_2}(u_i)\right| \left|\varphi f_{\tilde{A}_1}(u_i) - \varphi f_{\tilde{A}_2}(u_i)\right|)) \end{aligned}$$

Case 5: If we let $w_1 = w_2 = w_3 = w_4 = \frac{1}{3}$ and consider that every degree is important, then we can achieve the similarity degree [5] of $\tilde{A}_1, \tilde{A}_2$ as follows:

$$T_5(\tilde{A}_1, \tilde{A}_2) = \frac{1}{n}\sum_{i=1}^{n}(1-\frac{1}{3}(\left|\mu f_{\tilde{A}_1}(u_i) - \mu f_{\tilde{A}_2}(u_i)\right|$$

$$+\left|\nu f_{\tilde{A}_1}(u_i)-\nu f_{\tilde{A}_2}(u_i)\right|+\left|\pi f_{\tilde{A}_1}(u_i)-\pi f_{\tilde{A}_2}(u_i)\right|$$
$$+\left|\varphi f_{\tilde{A}_1}(u_i)-\varphi f_{\tilde{A}_2}(u_i)\right|))$$

Case 6: If we let $w_1 = w_2 = w_3 = \frac{1}{4}$, $w_4 = \frac{1}{2}$ and take all degrees into account, but focus more on the mean degree of membership, then we can get the similarity degree [6] of $\tilde{A}_1$, $\tilde{A}_2$ as follows:

$$\begin{aligned} T_6(\tilde{A}_1,\tilde{A}_2) &= \frac{1}{n}\sum_{i=1}^{n}\Bigg(1-\frac{1}{4}(\left|\mu f_{\tilde{A}_1}(u_i)-\mu f_{\tilde{A}_2}(u_i)\right| \\ &\quad +\left|\nu f_{\tilde{A}_1}(u_i)-\nu f_{\tilde{A}_2}(u_i)\right| \\ &\quad +\left|\pi f_{\tilde{A}_1}(u_i)-\pi f_{\tilde{A}_2}(u_i)\right|) \\ &\quad -\frac{\left|\varphi f_{\tilde{A}_1}(u_i)-\varphi f_{\tilde{A}_2}(u_i)\right|}{2}\Bigg) \end{aligned}$$

In the next definition, we will consider similarity classes based on the similarity degrees instead of intuitionistic fuzzy equivalence classes. Hence, we can get the similarity degree between x_i, x_j on an attribute a_k in IFDT by the following.

Definition 2. Let $\mathcal{I} = (U, A_1 \cup \{d\})$ be an intuitionistic fuzzy decision table. For any attribute $a_k \in A_1$, the similarity degrees between u_i, u_j on an attribute a_k can be denoted by the following.

Definition 3. For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, $A_2 \subseteq A_1$ and a similarity rate $\theta \in [0, 1]$, the θ-similarity class of an object $u_i \in U$ is defined as
$T^{\theta}_{lA_2}(u_i) = \{u_j | t^{a_k}_{lij} \geq \theta, \forall a_k \in A_2, u_j \in U\}$, $l = 1, \ldots, 6$. If $u_j \in T^{\theta}_{lA_2}(u_i)$, it can be found that the similarity degree of u_i and u_j according to any attribute in A_2 is not less than the given θ. That is to say, $T^{\theta}_{lA_2}(u_i)$ is the set of objects. This set can be indiscernible with the object u_i under the similarity rate θ associated with the attribute set A_2. In general, θ is the given similarity rate for an intuitionistic fuzzy decision table. The new similarity relation is denoted by $T^{\theta}_{lA_2}$.

Definition 4. Let $\mathcal{I} = (U, A_1 \cup \{d\})$ be an intuitionistic fuzzy decision table. $A_2 \subseteq A_1$, $X \subseteq U$, the upper approximation and lower approximation of Xon A_2, are described as follows:

$$\overline{T^{\theta}_{lA_2}}(X) = \{u \in U | T^{\theta}_{lA_2}(u) \cap X \neq \emptyset\};$$
$$\underline{T^{\theta}_{lA_2}}(X) = \{u \in U | T^{\theta}_{lA_2}(u) \subseteq X\}.$$
$$(l = 1, 2, 3, 4, 5, 6).$$

where $\overline{T^{\theta}_{lA_2}}(X)$ and $\underline{T^{\theta}_{lA_2}}(X)$ are known as the θ-similarity approximation operators in the intuitionistic fuzzy decision table. If $\underline{T^{\theta}_{lA_2}}(X) = \overline{T^{\theta}_{lA_2}}(X)$, then the set X is a definable set. Otherwise, the set X is a rough set with respect to A_2 in IFDT.

$$\begin{aligned}
t^{a_k}_{1ij} &= 1-\left|\varphi f_{a_k}(u_i)-\varphi f_{a_k}(u_j)\right|, \\
t^{a_k}_{2ij} &= 1-\frac{\left|\mu f_{a_k}(u_i)-\mu f_{a_k}(u_j)\right|+\left|\nu f_{a_k}(u_i)-\nu f_{a_k}(u_j)\right|}{2}, \\
t^{a_k}_{3ij} &= 1-\frac{1}{4}(\left|\mu f_{a_k}(u_i)-\mu f_{a_k}(u_j)\right| \\
&\quad +\left|\nu f_{a_k}(u_i)-\nu f_{a_k}(u_j)\right|)-\frac{1}{2}(\left|\varphi f_{a_k}(u_i)-\varphi f_{a_k}(u_j)\right|), \\
t^{a_k}_{4ij} &= 1-\frac{1}{3}(\left|\mu f_{a_k}(u_i)-\mu f_{a_k}(u_j)\right| \\
&\quad +\left|\nu f_{a_k}(u_i)-\nu f_{a_k}(u_j)\right|+\left|\varphi f_{a_k}(u_i)-\varphi f_{a_k}(u_j)\right|), \\
t^{a_k}_{5ij} &= 1-\frac{1}{3}(\left|\mu f_{a_k}(u_i)-\mu f_{a_k}(u_j)\right|+\left|\nu f_{a_k}(u_i)-\nu f_{a_k}(u_j)\right| \\
&\quad +\left|\pi f_{a_k}(u_i)-\pi f_{a_k}(u_j)\right|+\left|\varphi f_{a_k}(u_i)-\varphi f_{a_k}(u_j)\right|), \\
t^{a_k}_{6ij} &= 1-\frac{1}{4}(\left|\mu f_{a_k}(u_i)-\mu f_{a_k}(u_j)\right|+\left|\nu f_{a_k}(u_i)-\nu f_{a_k}(u_j)\right| \\
&\quad +\left|\pi f_{a_k}(u_i)-\pi f_{a_k}(u_j)\right|)-\frac{1}{2}(\left|\varphi f_{a_k}(u_i)-\varphi f_{a_k}(u_j)\right|)
\end{aligned}$$

Theorem 1. *Let $\mathcal{I} = (U, A_1 \cup \{d\})$ be an intuitionistic fuzzy decision table. $X, Y \subseteq U$, $A_2 \subseteq A_1$, $l = 1, \ldots, 6$, the following properties hold.*

(1) $\underline{T^{\theta}_{lA_2}}(X) \subseteq X \subseteq \overline{T^{\theta}_{lA_2}}(X)$,

(2) $\underline{T^{\theta}_{lA_2}}(X) = \overline{T^{\theta}_{lA_2}}(X)$, $\overline{T^{\theta}_{lA_2}}(X) = \underline{T^{\theta}_{lA_2}}(X)$,

(3) $\underline{T^{\theta}_{lA_2}}(\emptyset) = \overline{T^{\theta}_{lA_2}}(\emptyset) = \emptyset$, $\overline{T^{\theta}_{lA_2}}(U) = \underline{T^{\theta}_{lA_2}}(U) = U$,

(4) $\underline{T^{\theta}_{lA_2}}(X \cap Y) = \underline{T^{\theta}_{lA_2}}(X) \cap \underline{T^{\theta}_{lA_2}}(Y)$,
$\overline{T^{\theta}_{lA_2}}(X \cup Y) = \overline{T^{\theta}_{lA_2}}(X) \cup \overline{T^{\theta}_{lA_2}}(Y)$,

(5) $\underline{T^{\theta}_{lA_2}}(X \cap Y) \subseteq \underline{T^{\theta}_{lA_2}}(X) \cap \underline{T^{\theta}_{lA_2}}(Y)$,
$\overline{T^{\theta}_{lA_2}}(X \cup Y) \supseteq \overline{T^{\theta}_{lA_2}}(X) \cup \overline{T^{\theta}_{lA_2}}(Y)$,

(6) $X \subseteq Y \Rightarrow \underline{T^{\theta}_{lA_2}}(X) \subseteq \underline{T^{\theta}_{lA_2}}(Y)$,
$\overline{T^{\theta}_{lA_2}}(X) \subseteq \overline{T^{\theta}_{lA_2}}(Y)$,

(7) $\underline{T^{\theta}_{lA_2}}\left(\underline{T^{\theta}_{lA_2}}(X)\right) = \underline{T^{\theta}_{lA_2}}(X)$,
$\overline{T^{\theta}_{lA_2}}\left(\overline{T^{\theta}_{lA_2}}(X)\right) = \overline{T^{\theta}_{lA_2}}(X)$.

Proof. These properties can be acquired directly by Definition 4.

In the next theorem, a partial order can be obtained on all the similarity relations on U in an intuitionistic fuzzy decision table.

Theorem 2. *Let $\mathcal{I} = (U, A_1 \cup \{d\})$ be an intuitionistic fuzzy decision table. If $A_2, A_3 \subseteq A_1$, $A_3 \subseteq A_2$, $u_i \in U$, then the following property can be obtained*:

$$T^{\theta}_{lA_2}(u_i) \subseteq T^{\theta}_{lA_3}(u_i),\ l = 1, 2, 3, 4, 5, 6.$$

Proof. Specially, two special similarity relations can be found directly.

(1) $T^{\theta}_{lA_2}(u_i) = \{u_i\}$, then this is the finest relation.
(2) $T^{\theta}_{lA_2}(u_i) = U$, then this is the coarsest relation.

Theorem 3. *For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, if $A_2 \subseteq A_1$, $0 \leq \lambda \leq \theta \leq 1$, then we can have:*

$$T^{\theta}_{lA_2}(u_i) \subseteq T^{\lambda}_{lA_2}(u_i),\ l = 1, 2, 3, 4, 5, 6.$$

Additionally, we can also obtain $T^{\theta}_{lA_2}(u_i) = \bigcap_{a_k \in A_2} T^{\theta}_{l\{a_k\}}(u_i)$. Furthermore, $T^{\theta}_{lA_2}(u_i) \neq \emptyset$ and $\bigcup_{u_i \in U} T^{\theta}_{lA_2}(u_i) = U$.

Definition 5. For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, the decision class of an object $u_i \in U$ is denoted as

$$D(u_i) = \{u_j | d(u_i) = d(u_j)\}\ (u_j \in U).$$

Let U/d denote the partition induced by the decision attribute d.

The uncertainty measures will be discussed in the intuitionistic fuzzy decision table in the next section. In order to measure the uncertainty of an intuitionistic fuzzy decision table, we mainly use θ-conditional entropy, θ-similarity intuitionistic fuzzy accuracy, θ-similarity intuitionistic fuzzy roughness, and θ-rough decision entropy.

4. Some uncertainty measures in IFDT

Definition 6. For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, the θ-conditional entropy of d on the attribute set A_2 is defined by the following.

$$H^{\theta}_{lT}(d|A_2) = -\sum_{i=1}^{|U|}\sum_{j=1}^{|U/d|} \frac{\left|T^{\theta}_{lA_2}(u_i) \cap D_j\right|}{|U|^2} \log_2 \frac{\left|T^{\theta}_{lA_2}(u_i) \cap D_j\right|}{\left|T^{\theta}_{lA_2}(u_i)\right|}$$

$l = 1, \ldots, 6$

Theorem 4. *For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, and $A_2, A_3 \subseteq A_1$: If $\forall u_i \in U$, $T^{\theta}_{lA_2}(u_i) = T^{\theta}_{lA_3}(u_i)$, then*
$H^{\theta}_{lT}(d|A_2) = H^{\theta}_{lT}(d|A_3)\ (i = 1, 2, 3, 4, 5, 6)$.

Proof. Proof can be found in Definition 6.

Theorem 5. *For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, if $A_3 \subseteq A_2 \subseteq A_1$, then we have $H^{\theta}_{lT}(d|A_2) \leq H^{\theta}_{lT}(d|A_3)\ (i = 1, 2, 3, 4, 5, 6)$.*

Theorem 6. *For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, and $A_2 \subseteq A_1$ are two attribute sets, $\log |U|$ is the maximum conditional entropy of d with reference to A_2. Also,*

$$H^{\theta}_{lT}(d|A_2) = \log_2 |U|\ \ (l = 1, 2, 3, 4, 5, 6)$$

only if for any $u \in U$, $T^{\theta}_{lA_2}(u_i) = U$, and for any $D_j \in U/d$, $\left|D_j\right| = 1$.

Theorem 7. *For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, and when $A_2 \subseteq A_1$ are two attribute sets, 0 is the minimum conditional entropy of d with reference to A_2. Also, $H^{\theta}_{lT}(d|A_2) = 0$*

$(l = 1, \ldots, 6)$ *if and only if* $T^{\theta}_{lA_2}(u_i) \subseteq D(u_i)$ *for all* $u_i \in U$.

Theorem 8. *For intuitionistic fuzzy decision table* $\mathcal{I} = (U, A_1 \cup \{d\})$ *and* $A_2 \subseteq A_1$, *if* $0 \leq \theta_1 \leq \theta_2 \leq 1$, *then we have*

$$H^{\theta_2}_{lT}(d|A_2) \leq H^{\theta_1}_{lT}(d|A_2).$$

In rough set theory, uncertainty measure is a hot topic nowadays. When classifying objects under the given attribute subset, the percentage of possible precise decision can be shown by the approximation accuracy. Also, an intuitionistic fuzzy decision table can be extended by making use of the operators of lower approximation and upper approximation.

Definition 7. For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, and $A_2 \subseteq A_1$, the θ-similarity intuitionistic fuzzy accuracy and roughness with respect to decision attribute d can be described as follows.

$$\alpha^{\theta}_{lA_2}(U/d) = \frac{\sum_{D_i \in U/d} \left| \underline{T^{\theta}_{lA_2}}(D_i) \right|}{\sum_{D_i \in U/d} \left| \overline{T^{\theta}_{lA_2}}(D_i) \right|} \quad (l = 1, \ldots, 6),$$

$$\rho^{\theta}_{lA_2}(U/d) = 1 - \frac{\sum_{D_i \in U/d} \left| \underline{T^{\theta}_{lA_2}}(D_i) \right|}{\sum_{D_i \in U/d} \left| \overline{T^{\theta}_{lA_2}}(D_i) \right|} \quad (l = 1, \ldots, 6).$$

Theorem 9. *For an intuitionistic fuzzy decision table* $\mathcal{I} = (U, A_1 \cup \{d\})$, *and* $A_3 \subseteq A_2 \subseteq A_1$, *with a given similarity rate* θ, *then*

$$\alpha^{\theta}_{lA_2}(U/d) \geq \alpha^{\theta}_{lA_3}(U/d) \ (l = 1, 2, 3, 4, 5, 6),$$
$$\rho^{\theta}_{lA_2}(U/d) \leq \rho^{\theta}_{lA_3}(U/d) \ (l = 1, 2, 3, 4, 5, 6),$$

However, the θ-similarity intuitionistic fuzzy accuracy and roughness can't satisfy the needs of the reality in some cases. Thus, new measures are required. In this paper, in order to measure the precision of classification more effectively, we propose a new concept denoted θ-rough decision entropy with respect to the intuitionistic fuzzy decision table as an improved version of θ-similarity intuitionistic fuzzy accuracy and roughness.

Definition 8. For an intuitionistic fuzzy decision table $\mathcal{I} = (U, A_1 \cup \{d\})$, and $A_2 \subseteq A_1$, and a given similarity rate θ, the θ-rough decision entropy in the intuitionistic fuzzy decision table is found by:

$$DE^{\theta}_{l}(A_2) = \rho^{\theta}_{lA_2}(U/d) H^{\theta}_{lT}(d|A_2).$$

Theorem 10. *For an intuitionistic fuzzy decision table* $\mathcal{I} = (U, A_1 \cup \{d\})$, *and* $A_3 \subseteq A_2 \subseteq A_1$, *then* $DE^{\theta}_{l}(A_2) \leq DE^{\theta}_{l}(A_3)$.

From the above theorem, it can be concluded that the θ-rough decision entropy with respect to the intuitionistic fuzzy decision table decreases as A_2 becomes finer.

Note: The relationship between the four measures we have mentioned is as follows.

Case 1: The relationship between θ-similarity intuitionistic fuzzy accuracy and roughness with respect to decision attribute d.

$$\alpha^{\theta}_{lA_2}(U/d) = 1 - \rho^{\theta}_{lA_2}(U/d),$$
$$\rho^{\theta}_{lA_2}(U/d) = 1 - \alpha^{\theta}_{lA_2}(U/d)$$

Case 2: The relationship between θ-similarity intuitionistic fuzzy accuracy and θ-rough decision entropy.

$$DE^{\theta}_{l}(A_2) = (1 - \alpha^{\theta}_{lA_2}(U/d)) H^{\theta}_{lT}(d|A_2)$$

Case 3: The relationship between θ-conditional entropy and θ-rough decision entropy.

As $\rho^{\theta}_{lA_2}(U/d) = 1$, in other words, $\alpha^{\theta}_{lA_2}(U/d) = 0$, we have

$$DE^{\theta}_{l}(A_2) = H^{\theta}_{lT}(d|A_2)$$

5. Comparison

In the next section, we will compare the proposed intuitionistic fuzzy clustering method with the existing methods. In order to facilitate comparison, the data in Table 1 are used to classify. There are 8 condition attributes and 20 objects in the intuitionistic fuzzy decision table shown in Table 1.

We put forward six kinds of similarity, which give six kinds of classification results. Take the first case as an example. Let $\theta = 0.6$, $A_2 = A_1$, the classification results are as follows:

$$T^{\theta}_{1A_1}(u_1) = \{u_1, u_3, u_4, u_6, u_7, u_{10}, u_{15}, u_{18}\};$$
$$T^{\theta}_{1A_1}(u_2) = \{u_2\}; \ T^{\theta}_{1A_1}(u_3) = \{u_1, u_3, u_6, u_8, u_9\};$$
$$T^{\theta}_{1A_1}(u_4) = \{u_4\}; \ T^{\theta}_{1A_1}(u_5) = \{u_5\};$$
$$T^{\theta}_{1A_1}(u_6) = \{u_6, u_7\};$$

Table 1
An intuitionistic fuzzy decision table

U	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	d
x_1	(0.8,0.2)	(0.2,0.7)	(0.3,0.6)	(0.5,0.5)	(0.1,0.9)	(0.2,0.5)	(0.8,0.1)	(0.6,0.3)	1
x_2	(0.5,0.3)	(0.2,0.7)	(0.8,0.1)	(0.2,0.5)	(0.6,0.3)	(0.4,0.5)	(0.2,0.3)	(1.0,0.0)	3
x_3	(1.0,0.0)	(0.3,0.6)	(0.6,0.2)	(0.3,0.3)	(0.3,0.5)	(0.8,0.1)	(0.3,0.4)	(0.6,0.3)	3
x_4	(0.6,0.3)	(0.1,0.8)	(0.7,0.1)	(0.6,0.3)	(0.6,0.3)	(0.6,0.3)	(0.4,0.4)	(0.3,0.7)	3
x_5	(0.8,0.2)	(0.4,0.5)	(0.8,0.0)	(0.3,0.4)	(0.6,0.4)	(0.7,0.2)	(0.4,0.2)	(0.2,0.8)	4
x_6	(0.8,0.1)	(0.1,0.8)	(0.7,0.1)	(0.2,0.4)	(0.5,0.3)	(0.4,0.3)	(0.8,0.2)	(0.1,0.8)	4
x_7	(0.7,0.1)	(0.7,0.1)	(0.0,0.9)	(0.5,0.4)	(0.5,0.1)	(0.1,0.7)	(0.5,0.4)	(0.2,0.8)	3
x_8	(0.9,0.0)	(0.6,0.3)	(0.2,0.6)	(0.3,0.5)	(0.8,0.2)	(0.7,0.2)	(0.6,0.3)	(0.3,0.7)	3
x_9	(0.1,0.8)	(0.3,0.6)	(0.1,0.8)	(0.5,0.4)	(0.8,0.1)	(0.6,0.3)	(0.7,0.2)	(0.7,0.3)	4
x_{10}	(0.5,0.4)	(0.1,0.7)	(0.1,0.8)	(0.1,0.4)	(0.4,0.3)	(0.9,0.1)	(0.2,0.1)	(0.6,0.4)	3
x_{11}	(0.2,0.8)	(0.8,0.2)	(0.7,0.2)	(0.4,0.5)	(0.4,0.2)	(0.8,0.2)	(0.4,0.3)	(0.5,0.5)	4
x_{12}	(0.7,0.1)	(0.2,0.8)	(0.8,0.1)	(0.4,0.6)	(0.6,0.3)	(0.4,0.6)	(0.2,0.4)	(0.4,0.6)	2
x_{13}	(0.6,0.3)	(0.5,0.4)	(0.9,0.0)	(0.6,0.3)	(0.3,0.6)	(0.5,0.3)	(0.6,0.3)	(0.5,0.3)	2
x_{14}	(0.7,0.1)	(0.2,0.8)	(0.7,0.2)	(0.4,0.5)	(0.2,0.7)	(0.2,0.8)	(0.3,0.6)	(0.3,0.7)	2
x_{15}	(0.4,0.5)	(0.2,0.7)	(0.9,0.0)	(0.3,0.5)	(0.6,0.4)	(0.3,0.7)	(0.9,0.1)	(0.6,0.4)	4
x_{16}	(0.1,0.8)	(0.6,0.3)	(0.6,0.2)	(0.3,0.7)	(0.5,0.3)	(0.6,0.4)	(0.8,0.1)	(0.6,0.3)	3
x_{17}	(0.8,0.1)	(0.5,0.1)	(0.8,0.1)	(0.1,0.9)	(0.2,0.4)	(0.4,0.5)	(0.7,0.3)	(0.3,0.7)	3
x_{18}	(0.2,0.5)	(0.6,0.3)	(0.8,0.1)	(0.9,0.1)	(0.3,0.7)	(0.6,0.4)	(0.6,0.4)	(0.8,0.2)	4
x_{19}	(0.3,0.4)	(0.1,0.9)	(0.7,0.2)	(0.6,0.2)	(0.3,0.4)	(0.2,0.5)	(0.5,0.5)	(0.5,0.2)	1
x_{20}	(0.6,0.3)	(0.2,0.8)	(0.8,0.1)	(0.7,0.1)	(0.5,0.4)	(0.5,0.2)	(0.3,0.7)	(0.3,0.6)	3

$$T^{\theta}_{1A_1}(u_7) = \{u_6, u_7, u_{13}, u_{14}, u_{19}\};$$

$$T^{\theta}_{1A_1}(u_8) = \{u_3, u_8, u_9, u_{11}, u_{16}\};$$

$$T^{\theta}_{1A_1}(u_9) = \{u_3, u_8, u_9, u_{10}, u_{11}, u_{13}, u_{14}, u_{17}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{10}) = \{u_1, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\}$$

$$T^{\theta}_{1A_1}(u_{11}) = \{u_8, u_9, u_{10}, u_{11}, u_{12}, u_{19}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{12}) = \{u_7, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{18}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{13}) = \{u_7, u_9, u_{10}, u_{12}, u_{13}\};$$

$$T^{\theta}_{1A_1}(u_{14}) = \{u_7, u_9, u_{10}, u_{14}, u_{15}, u_{16}, u_{18}\};$$

$$T^{\theta}_{1A_1}(u_{15}) = \{u_1, u_{10}, u_{12}, u_{14}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{16}) = \{u_8, u_{10}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}\};$$

$$T^{\theta}_{1A_1}(u_{17}) = \{u_9, u_{10}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{18}) = \{u_1, u_{10}, u_{12}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}\};$$

$$T^{\theta}_{1A_1}(u_{19}) = \{u_7, u_{10}, u_{11}, u_{15}, u_{17}, u_{18}, u_{19}, u_{20}\};$$

$$T^{\theta}_{1A_1}(u_{20}) = \{u_{10}, u_{11}, u_{12}, u_{15}, u_{17}, u_{19}, u_{20}\}.$$

Moreover, we consider the approach presented in [10] to obtain the following classification results:

$$[u_1]_{A_1} = \{u_1, u_3, u_4, u_6, u_7, u_{10}, u_{15}, u_{16}, u_{18}, u_{19}\};$$

$$[u_2]_{A_1} = \{u_2\};\ [u_3]_{A_1} = \{u_1, u_3, u_6, u_8, u_9\};$$

$$[u_4]_{A_1} = \{u_4\};$$

$$[u_5]_{A_1} = \{u_5\};\ [u_6]_{A_1} = \{u_6, u_7\};$$

$$[u_7]_{A_1} = \{u_6, u_7, u_{12}, u_{13}, u_{14}, u_{19}, u_{20}\};$$

$$[u_8]_{A_1} = \{u_3, u_8, u_9, u_{11}, u_{16}, u_{17}\};$$

$$[u_9]_{A_1} = \{u_3, u_8, u_9, u_{10}, u_{11}, u_{13}, u_{14}, u_{17}, u_{20}\};$$

$$[u_{10}]_{A_1} = \{u_1, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}\};$$

$$[u_{11}]_{A_1} = \{u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{19}, u_{20}\};$$

$$[u_{12}]_{A_1} = \{u_7, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{18}, u_{20}\};$$

$$[u_{13}]_{A_1} = \{u_7, u_9, u_{10}, u_{11}, u_{12}, u_{13}\};$$

$$[u_{14}]_{A_1} = \{u_7, u_9, u_{10}, u_{11}, u_{14}, u_{15}, u_{16}, u_{18}\};$$

$$[u_{15}]_{A_1} = \{u_1, u_{10}, u_{12}, u_{14}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\};$$

$[u_{16}]_{A_1} = \{u_1, u_8, u_{10}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}\};$

$[u_{17}]_{A_1} = \{u_8, u_9, u_{10}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\};$

$[u_{18}]_{A_1} = \{u_1, u_{10}, u_{12}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}\};$

$[u_{19}]_{A_1} = \{u_1, u_7, u_{10}, u_{11}, u_{15}, u_{17}, u_{18}, u_{19}, u_{20}\};$

$[u_{20}]_{A_1} = \{u_7, u_{11}, u_{12}, u_{15}, u_{17}, u_{19}, u_{20}\}.$

From the above results, we can see that the clustering method and the method proposed in article [10] are applications of the intuitionistic fuzzy similarity formula of different schemes for clustering, and the clustering results have little difference, which shows that this method is effective. Moreover, although our method is effective compared with the traditional fuzzy clustering method, our method does not have a significant advantage in computational complexity. When the number of objects and attributes of the data set exceeds a certain number, the proposed method appears to have no advantage.

6. Numerical calculation

In the next, in order to verify and test the validity and effectiveness of the given uncertainty measure, numerical experiments are performed on different intuitionistic fuzzy decision tables. The intuitionistic fuzzy decision table is represented in Table 1.

Let $\theta = 0.6$, $A_2 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$, $A_2 \subseteq A_1$. There are six kinds of similarity degree between two intuitionistic fuzzy sets A_1, A_2 on U. We can calculate the similarity degree between any two IFs after computing the similarity degree; we can work out a θ-similarity class according to Definition 3. Finally, according to Definitions 6, 7 and 8, we can work out the accuracy and roughness, θ-conditional entropy and θ-rough decision entropy. The results of the accuracy and roughness,

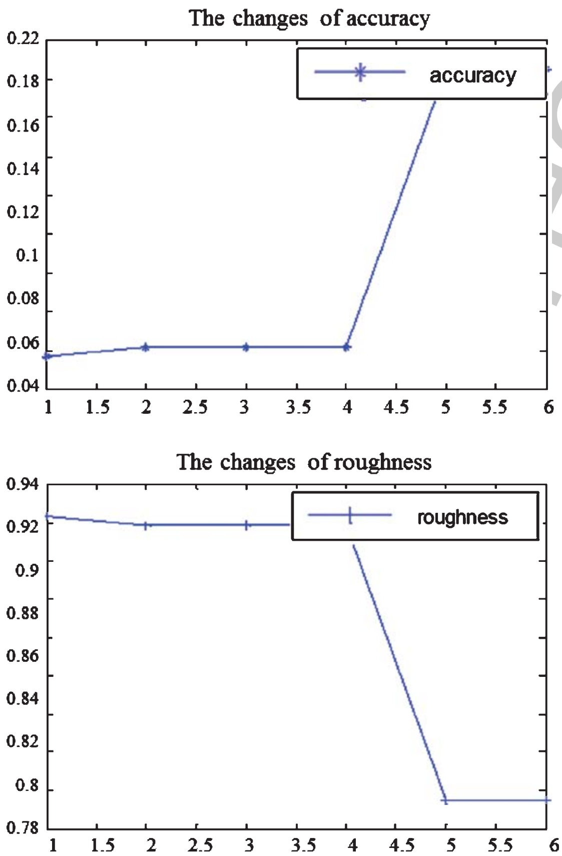

Fig. 1. The results of $\alpha_{IB}^{0.6}$, $\rho_{IB}^{0.6}$.

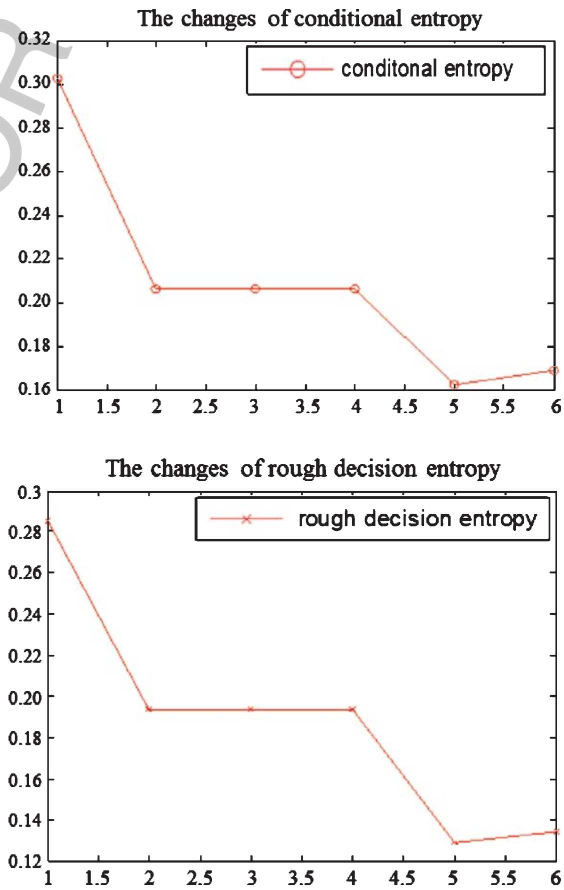

Fig. 2. The results of $H_{IS}^{0.6}(d|B)$ and $DE_I^{0.6}(B)$.

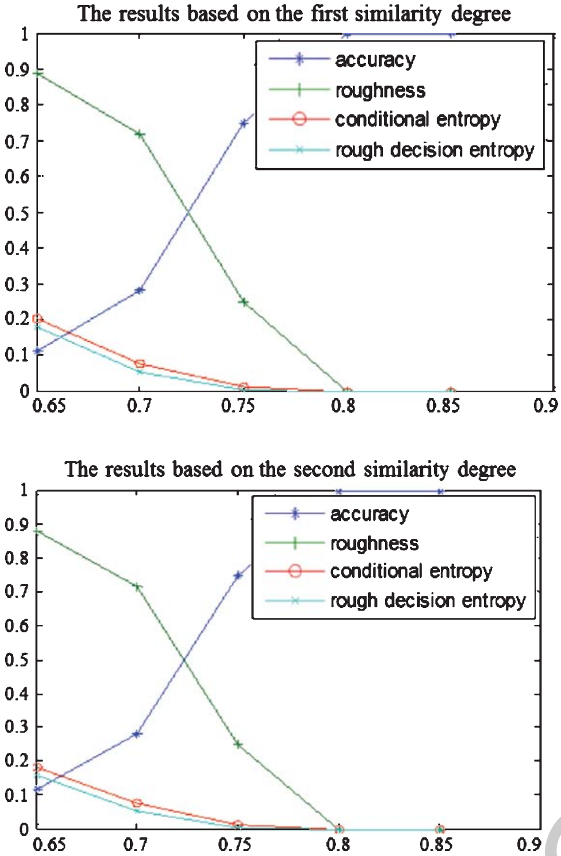

Fig. 3. The results based on the first and second similarity degrees.

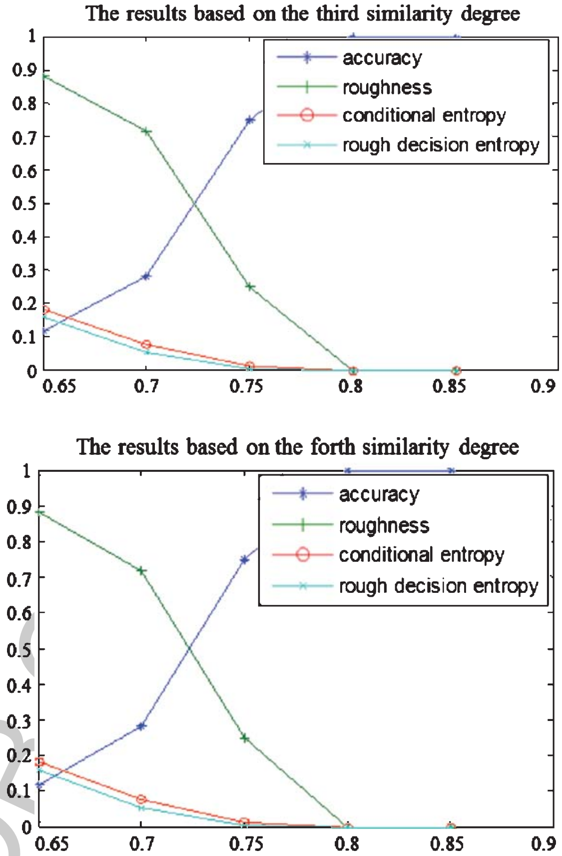

Fig. 4. The results based on the third and fourth similarity degrees.

θ-conditional entropy and θ-rough decision entropy based on the above six similarity degrees can be depicted in the following.

From Figs. 1 and 2, we can know that the accuracy is increasing and the roughness is decreasing when the similarity degree is changing from the first to the fifth. Also, the θ-condition and rough decision entropy are decreasing when the similarity degree is changing from the first to the fifth apart from the sixth. The minimum θ-condition entropy and θ-rough decision entropy can be acquired by using the fifth similarity degree. This is very consistent with the reality of life through the above numerical experiments, which can verify the validity and correctness of our definition.

In the next section, we will compare the changes of the accuracy, roughness, θ-conditional entropy and θ-rough decision entropy with a different similarity rate θ. The following pictures represent the results of six different similarity degrees. Let $A_2 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ be the condition attribute set; the results are shown in Figs. 3, 4, and 5.

We can obtain that the accuracies all are decreasing as the similarity rate θ decreases; the roughness, θ-conditional entropies and θ-rough decision entropies all are decreasing with the similarity rate θ decreasing. Furthermore, the second, third and fourth similarity degrees are similar. If we observe the pictures carefully, we will find that the accuracies and roughness are the same when $\theta = 0.65$ and $\theta = 0.7$ based on the fifth similarity degree, but their θ-conditional entropies and θ-rough decisions are different. Thus, we can know that the new measures are superior, and the differences are quite bvious when θ took different values. Additionally, the fifth classification accuracy is the highest based on the same similarity rates. Through comparing the changes of the accuracy, roughness, θ-conditional entropy and θ-rough decision entropy with the different similarity rates θ, we can obtain the correctness of our definition more closely, so that we can provide an effective method for the division of the intuitionistic fuzzy information system and the uncertainty measurement.

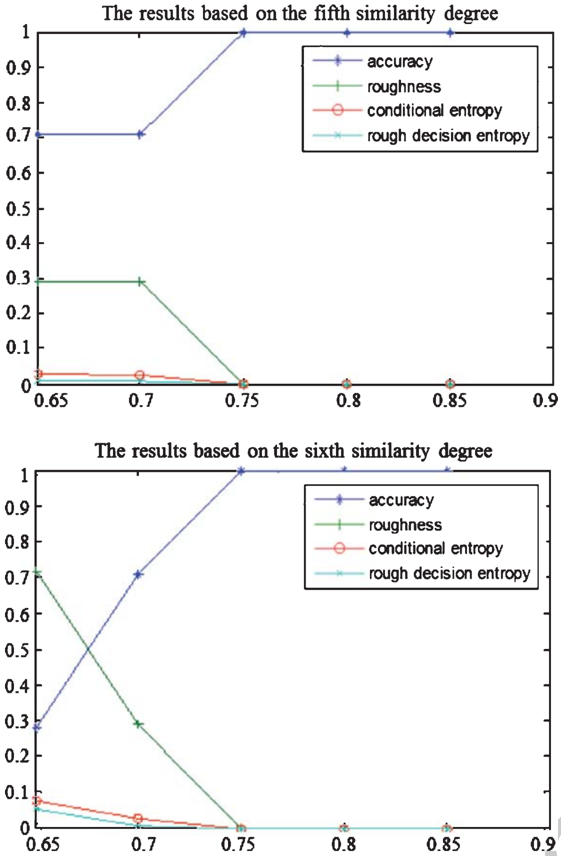

Fig. 5. The results based on the fifth and sixth similarity degrees.

7. Conclusions

In this paper, six similarity degrees of intuitionistic fuzzy sets were considered in order to measure the uncertainty of the intuitionistic fuzzy decision table. Also, we have given new classification methods based on different similarity degrees in this information table. Furthermore, we discussed the upper approximation and lower approximation of a given set and investigated their properties in an intuitionistic fuzzy table. In addition, we proposed the extended θ-conditional entropy and θ-rough decision entropy in an intuitionistic fuzzy decision table with respect to similarity measures of intuitionistic fuzzy values. What's more, we deeply explored their properties. Finally, we compared the accuracies, roughness, θ-conditional entropies and θ-rough decision entropies based on different similarity measures and similarity rates by conducting the experiments. The classical decision classes are used in intuitionistic fuzzy decision tables and the issues about fuzzy or intuitionistic fuzzy decision classes will be our next works.

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